

## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <a href="http://about.jstor.org/participate-jstor/individuals/early-journal-content">http://about.jstor.org/participate-jstor/individuals/early-journal-content</a>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

As a matter of interest, the growth during 1920 has been estimated on the basis of actual immigration and emigration data and projected births and deaths in the registration states. This results, as shown also in Table II, in an estimated population of 107,320,278 for January 1, 1921. It is true, of course, that in applying this method to projections of future growth no information as to the trend of the ratio between the population of the selected registration states and that of the United States is available. The error involved in projecting the ratios found in the 1910–20 decade will probably not be appreciable.

The accompanying chart shows the population estimates for the years 1910 to 1921 inclusive, and a straight-line projection of the 1900–10 growth.

## A SIMPLE GRAPHIC CONSTRUCTION FOR FARR'S RELATION BETWEEN BIRTH-RATE, DEATH-RATE, AND MEAN LENGTH OF LIFE\*

By Alfred J. Lotka

In a previous article† the writer has discussed the rational basis of Farr's rule‡

$$\frac{1}{3} \cdot \frac{1}{b} + \frac{2}{3} \cdot \frac{1}{d} = L \tag{1}$$

where

$$b = birth$$
-rate per head (2)

$$d = \text{death-rate per head}$$
 (3)

$$L = \text{mean length of life.}$$
 (4)

Writing Farr's rule in the more general form

$$P\frac{1}{b} + Q\frac{1}{d} = L \tag{5}$$

or

$$p\frac{1}{b} + q\frac{1}{d} = 1\tag{6}$$

$$(b-p)(d-q) = pq \tag{7}$$

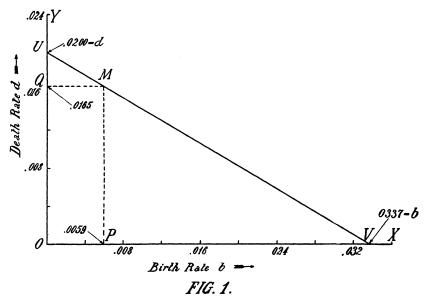
it was shown in the paper cited how the values of P, Q (represented in Farr's formula by the approximate numerical coefficients  $\frac{1}{3}$  and  $\frac{2}{3}$ ) could be determined exactly.

<sup>\*</sup>Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health Johns Hopkins University. No. 46.

<sup>†</sup> Quart. Pub. Am. Statis. Assn., Sept., 1918, p. 121.

<sup>‡</sup> Quoted by Newsholme, Vital Statistics, 1899, p. 301.

In the course of some work on malaria epidemiology now in progress and to be published elsewhere later, a somewhat similar relation between certain variables was encountered, leading to the following graphic construction.



Along 
$$OX$$
 mark off a length  $OP = p = \frac{P}{L}$  (8)

Along 
$$OY$$
 mark off a length  $OQ = q = \frac{Q}{L}$  (9)

Complete the rectangle QOPM

Suppose we are now given

$$b = 0.0337 \tag{10}$$

It is required to find d

Along 
$$OX$$
 mark off  $OV = 0.0337$  (11)

Join VM and produce it to meet OY in U

Read off at U on the scale of OY

$$d = 0.0200. (12)$$

The figures employed in this example are those for the female population of England and Wales 1871-80, a period during which (as the writer has previously shown) the observed figures for many characteristics of the population agree very closely with those computed for a "Natural Population Norm."\* In the particular instance here chosen

<sup>\*</sup> Jour. Washington Acad. Sci., vol. 3, May, 1913, pp. 241-89.

P = 0.2631 (13)

$$Q = 0.7369$$
 (14)

$$L = 44.62$$
 (15)

$$\frac{P}{L} = 0.01651\tag{16}$$

$$\frac{Q}{L} = 0.00590 \tag{17}$$

i. e.,

$$0.2631 \frac{1}{b} + 0.7369 \frac{1}{d} = 44.62. \tag{18}$$

Putting 
$$b = 0.03373$$
 (observed) we find (19)

$$d = 0.02000 \text{ (calculated)} \tag{20}$$

as against

$$d = 0.020001$$
 (observed). (21)

I take this opportunity thus to supplement the data given for the male population in my previous paper. At the time it was written the extraordinarily close agreement between calculated and observed figures for the female population had escaped my notice.

## A VECTOR METHOD FOR COMPUTING A WEIGHTED AVERAGE

By R. von Huhn

In terms of dynamics the arithmetic mean is the center of gravity of a distribution. To illustrate this fact we may assume that certain values represent unit weights which are suspended at intervals upon a massless rod. The ideal rod which is under the action of these weights or external forces is at rest or in static equilibrium when the forces balance each other. The location of the fulcrum is therefore the position of the average with respect to the weights.

As the weights or forces have both magnitude and direction (parallel in this case), the two essentials that are necessary for the treatment of data by vector analysis are given.

Suppose we have at hand the statistical data as shown in the following table: